Modeling Nash Equilibria in an Electricity Market

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Primary Reference is

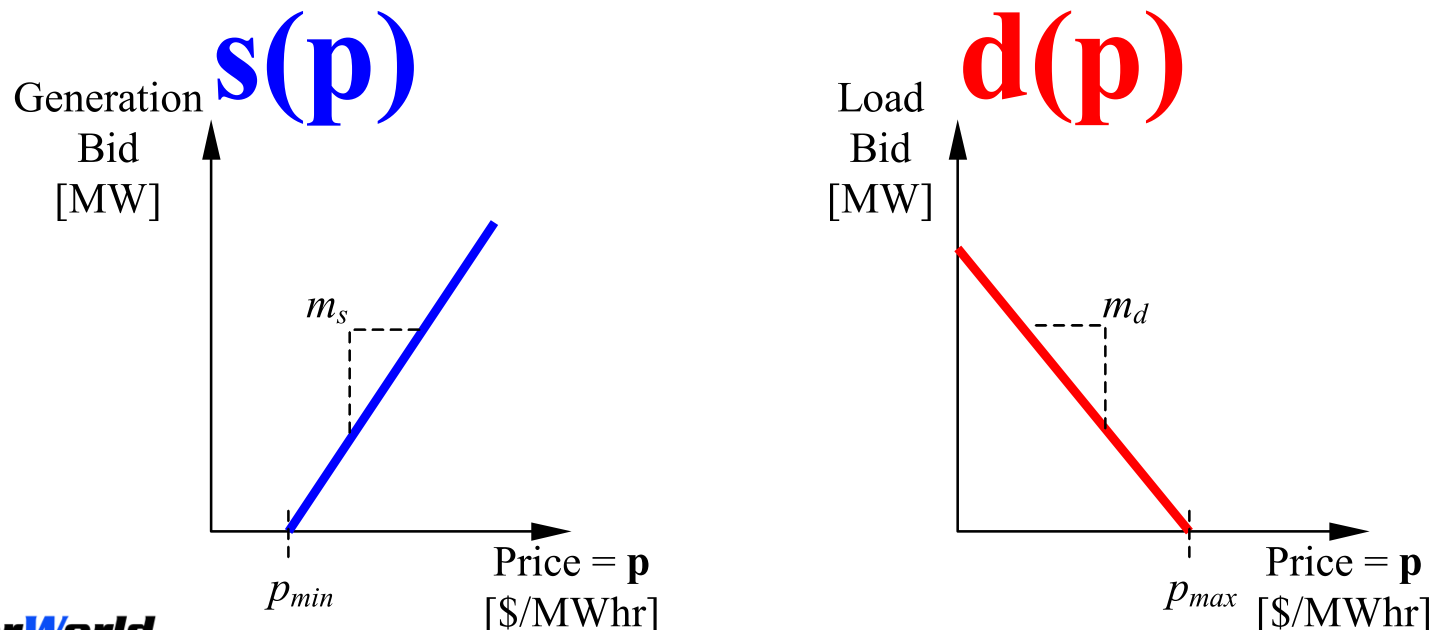
J.D. Weber and T.J. Overbye "An Individual Welfare Maximization Algorithm for Electricity Markets," *IEEE Transactions on Power Systems*, vol. 17, no. 3, August 2002, pp. 590-596.

Electricity Market Model

- Market participants (individuals) will consist of generator and loads submitting bids into the market
- Market will be cleared using an OPF or SCOPF solution
- All individuals will receive (or pay) the price at their market node.

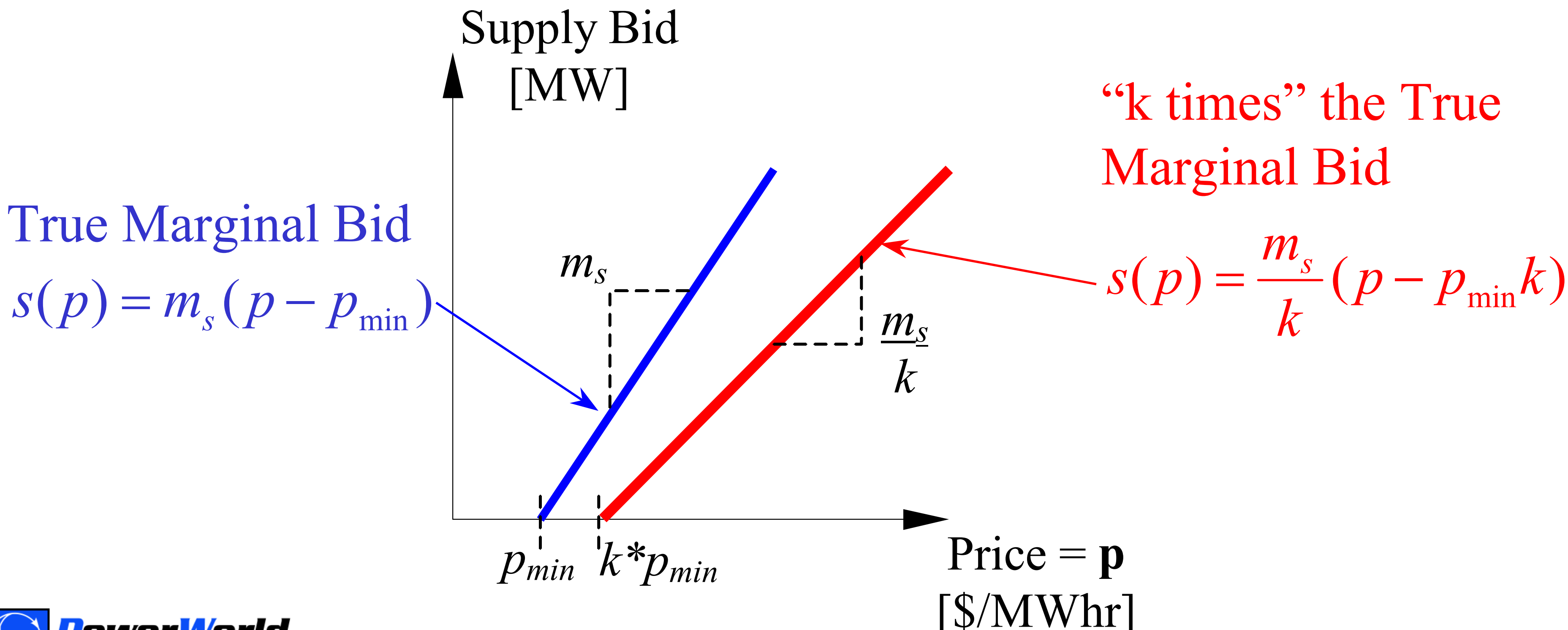
Market Bid Setup

- Suppliers and Consumers submit generation and load bids
  - For given price, submit a generation or load level



We will vary Market bids: Limit Possible Bids to Linear

- Each supplier chooses some ratio above or below its true marginal cost function



What does an Individual Want?

- Individual knows the method used to calculate its price and dispatch
  - An OPF or SCOPF will be solved
- Individual has some idea, based on past history, what its opponents are likely to bid
  - Make an assumption about their bids
- Using this information, an individual wants to determine a bid that will maximize its overall individual welfare

$$f(\mathbf{s},\mathbf{d},\lambda) = \sum_{\substack{\text{i=controlled}\\ \text{demands}}} [\underbrace{B_i(d_i)}_{\text{Benefits}} - \underbrace{\lambda_i \mathrm{d}_i}_{\text{Expenses}}] + \sum_{\substack{\text{controlled}\\ \text{supplies}}} [-\underbrace{C_i(s_i)}_{\text{Costs}} + \underbrace{\lambda_i \mathrm{s}_i}_{\text{Revenues}}]$$

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Algorithm for determining a Best Response in this Market Structure

- A "Nested Optimization Problem"

Individual's Welfare

$$\max_{\mathbf{k}} \quad f(\mathbf{s},\mathbf{d},\lambda)$$

s,d,λ are implicit functions of **k**

$$\text{s.t.} \quad (\mathbf{s},\mathbf{d},\lambda) \text{ are determined by } \underbrace{\begin{pmatrix} \text{OPF or SCOPF Solution} \\ \text{where "k bids" are variables} \end{pmatrix}}_{\text{"OPF Sub-Problem"}}$$

OPF Problem is a constraint" now

Market with Multiple Individuals

- Now consider a market with multiple market place participants (individuals)
- Assume they are all trying to maximum their welfare and determine optimal bids in the manner
- What will the market response be at a *steady state*?

Economic Market Equilibriums: The Nash Equilibrium

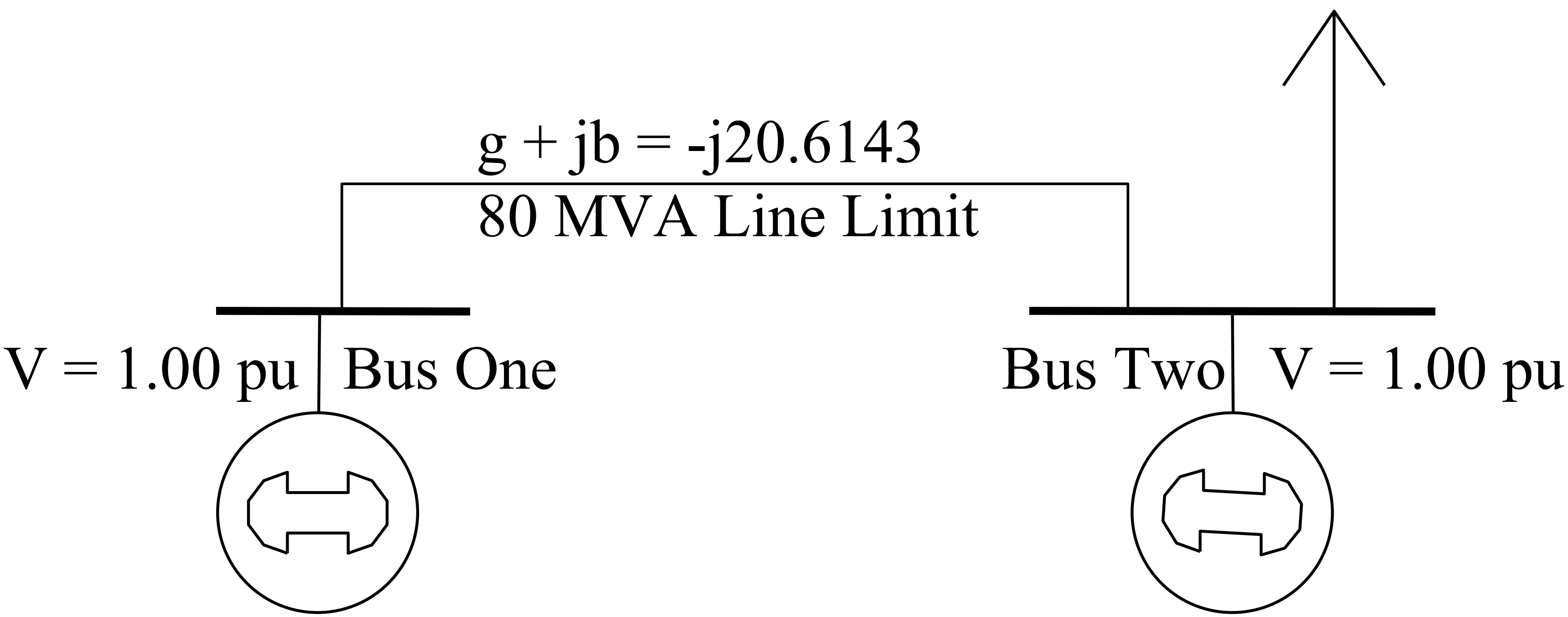
- Definition of a Nash Equilibrium
  - An individual looks at what its competitors are presently doing
  - The individual's best response to competitors' behavior is to continue its present behavior
  - This is true for ALL individuals in the market
- This is a Nash Equilibrium

Iterate the Nested Optimization Problem to find the Equilibrium

- Start all individuals at bids of $k = 1$
- Run the nested optimization for each individual and set its bid to its "best response"
- Continue running this optimization until the individuals stop changing their bids
- This will be a pure strategy Nash Equilibrium
  - Pure strategy: each bidder bids the same all the time

Simple Two Bus Example with Three Individuals

$$B_2(d_2) = k_{d2}\left(-0.04d_2^{\ 2} + 30d_2\right)$$

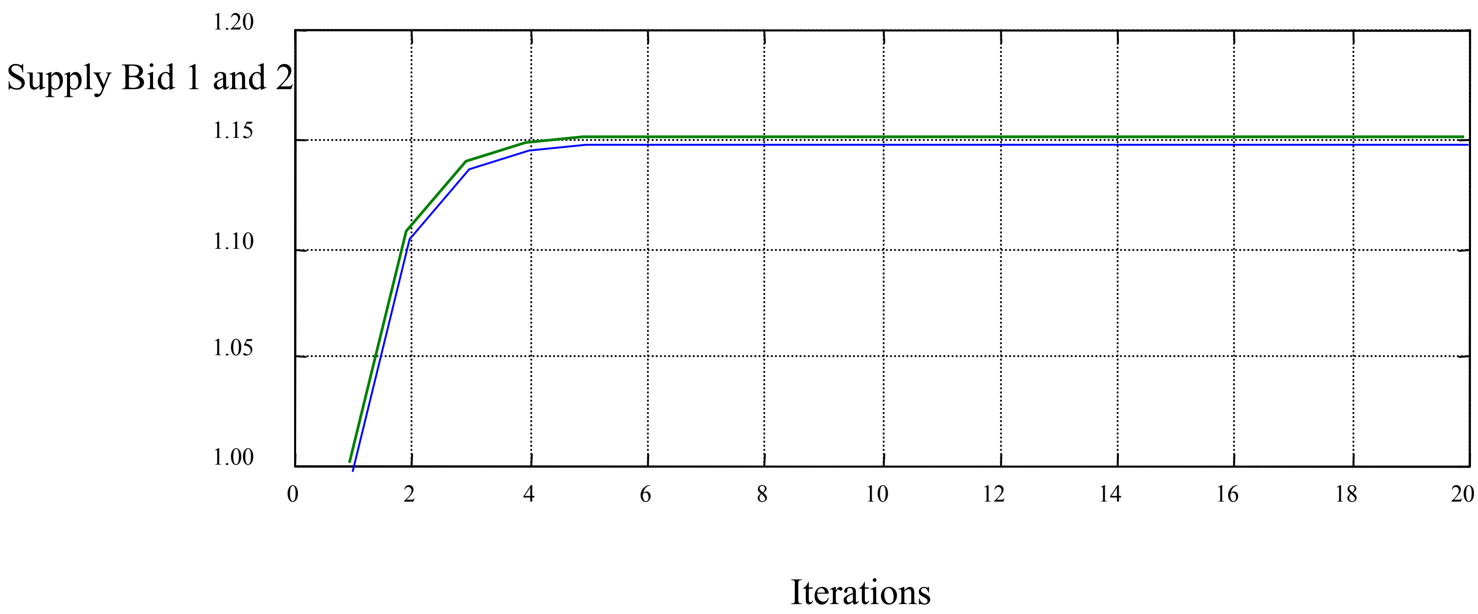


$$C_1(s_1) = k_{s1}\left(0.01s_1^{\ 2} + 10s_1\right) \qquad C_2(s_2) = k_{s2}\left(0.01s_2^{\ 2} + 10s_2\right)$$

forward

Consider Both Supplies "Competing" with NO Line Limit

- Set kd=1.00, then run competition
- Results: k_{g1} = 1.1502 and k_{g2} = 1.1502



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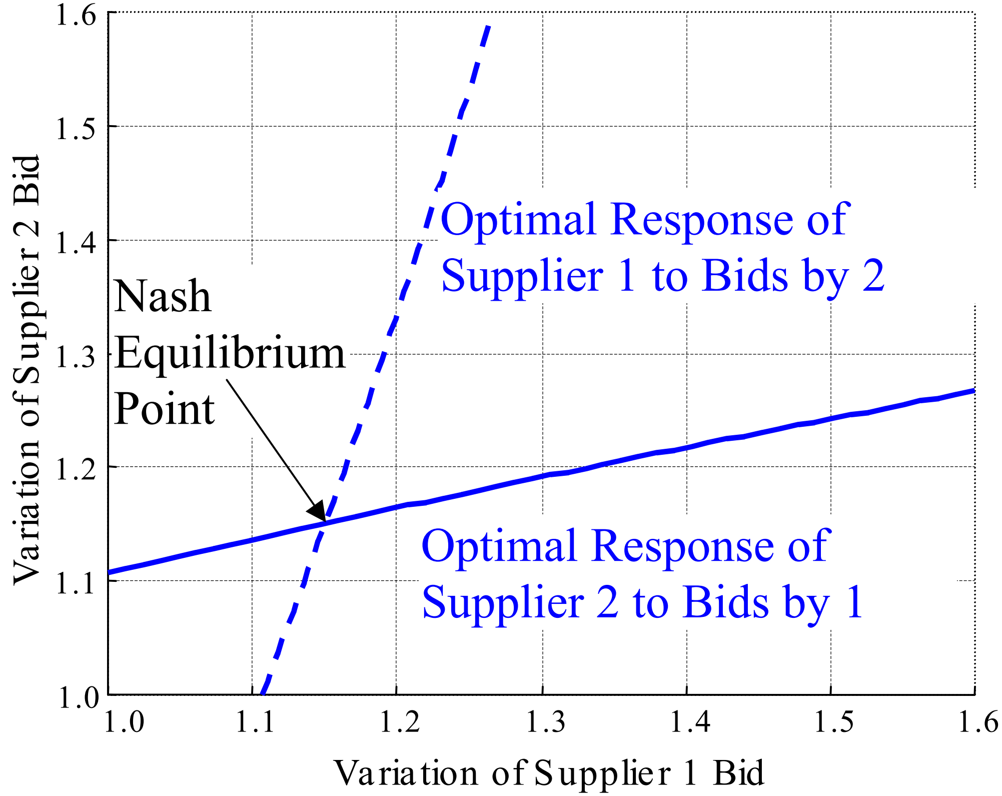
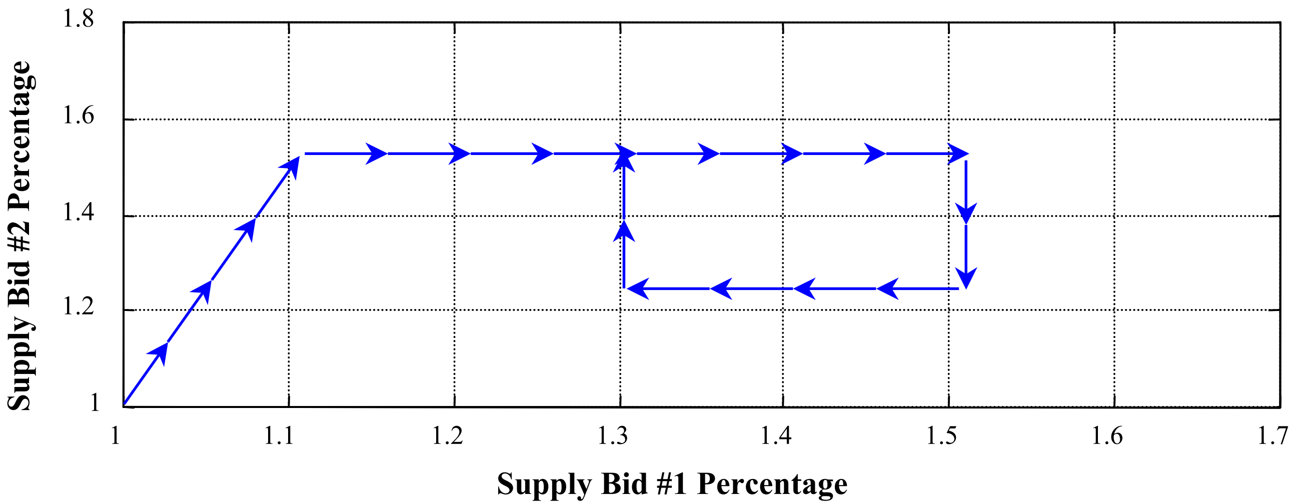
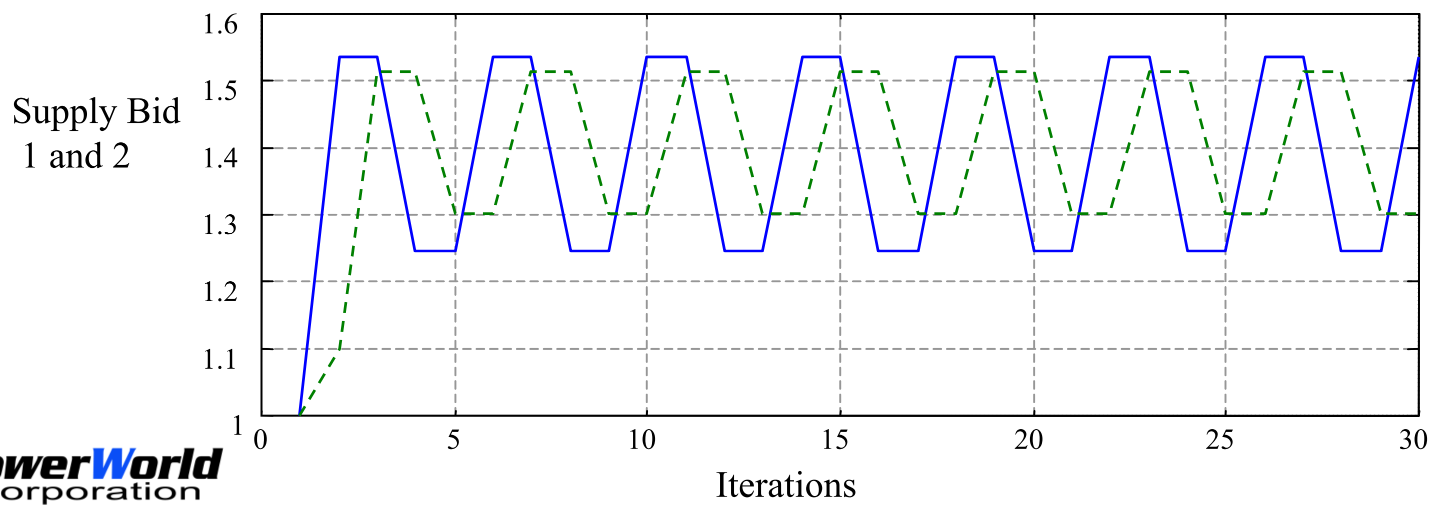
A Graphical Look at Nash Equilibrium in Two Dimensions

- Nash Equilibria are where the Optimal Response curves meet



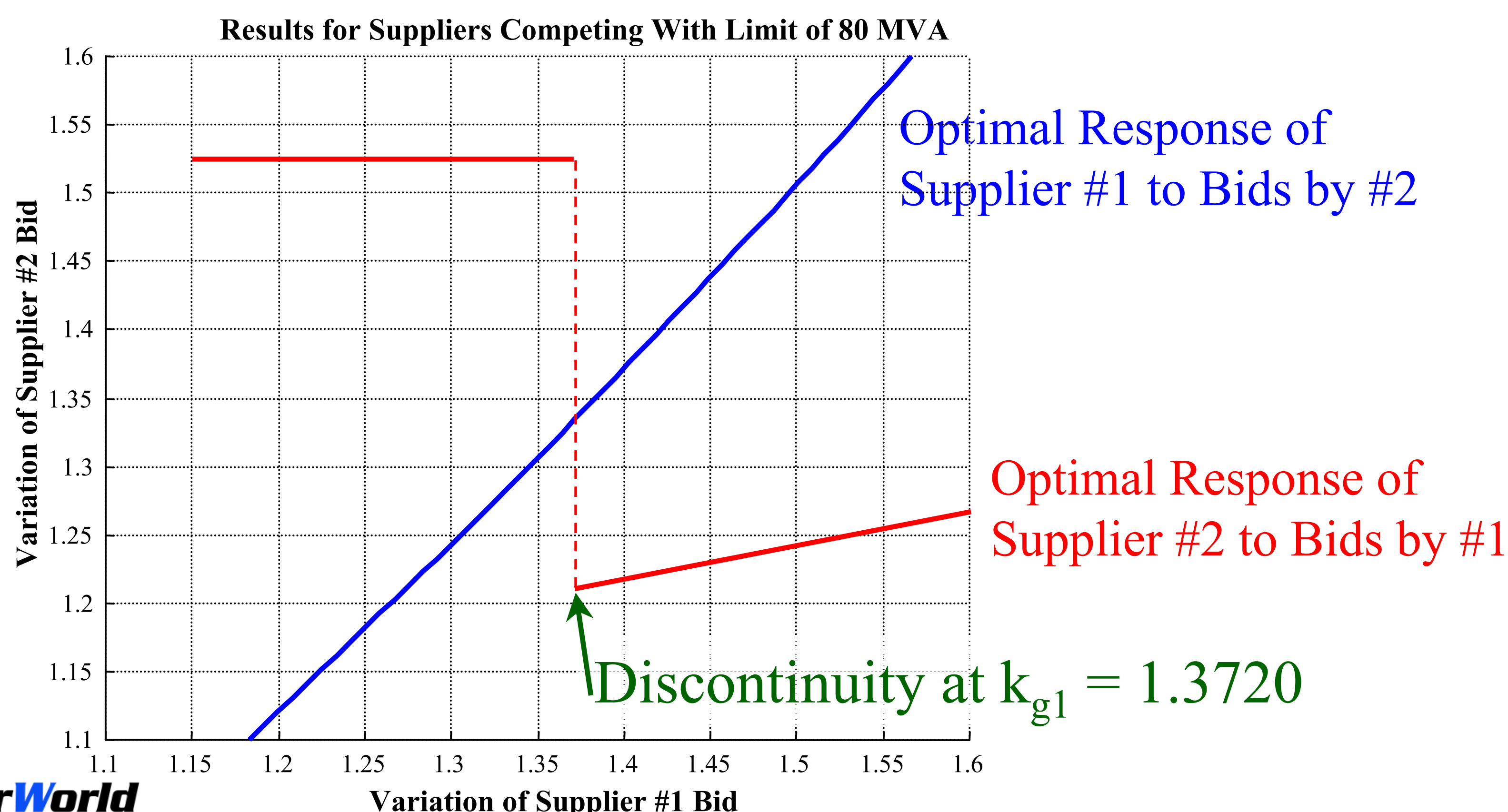
Results for Both Supplies Competing with an 80 MVA Line Limit





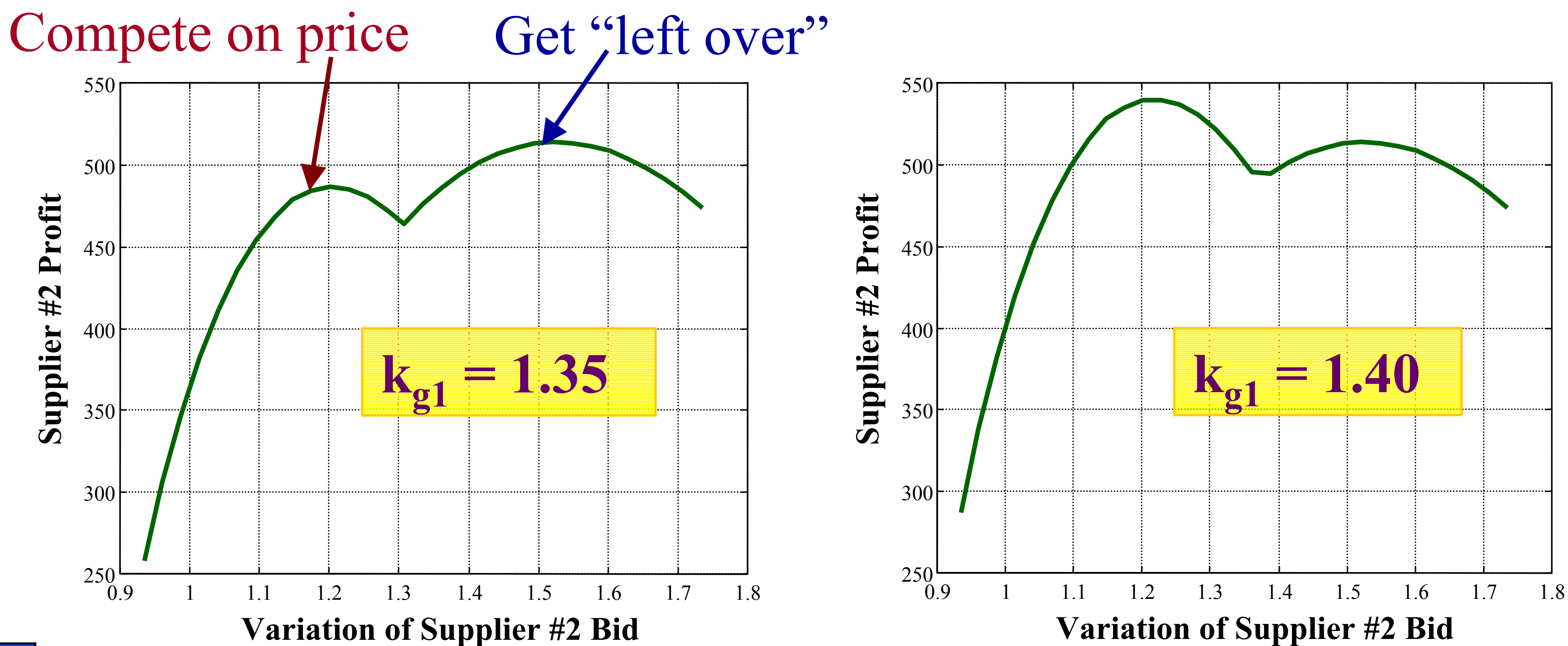
What's going on here?

- Optimal Curves Never Meet! No Equilibrium



Discontinuous Optimal Response?? Caused by Local Maxima

- Supplier #2 Profit Curves for values of k_{g1} on either side of discontinuous point



Does an Equilibrium Exist?

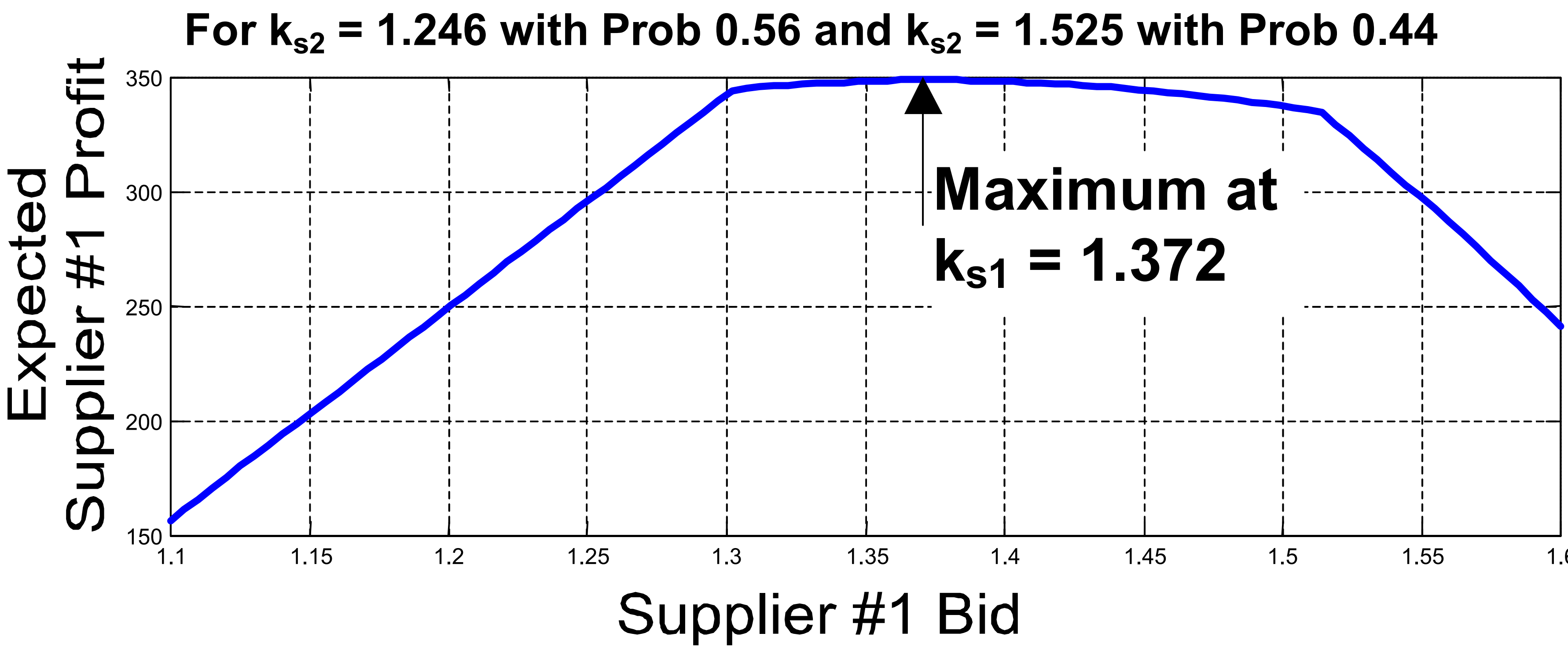
- We are only considering "pure" strategy
  - Only have shown that no pure strategy exist
- What are "mixed" strategy?
  - An individual chooses several pure strategies and assigns a probability to each.
  - The individual then submits these pure strategy according to their probability
- By including mixed strategy, a simple equilibrium is seen for this example

Mixed Strategy Nash Equilibrium

- Supplier #1: Bid
  - $k_{s1} = 1.372$ always
- Supplier #2: Bid
  - $k_{s2} = 1.246$ with Probability 0.56
  - $k_{s2} = 1.525$ with Probability 0.44
- For supplier #2:
  - Best response because when supplier #1 bids 1.372, supplier #2 has no preference between the two bids shown. Arbitrary probabilities are fine

Supplier #1: Expected Profit

- Expected Profit is maximized at $k_{s1} = 1.372$



Conclusions from Two-Bus Example

- Constraints can eliminate “pure” equilibrium
- Calculus-based method can not generally find more than one local optima, but …
  - Human experience will guide the algorithm user to constraints which can be gamed
  - Still useful for multiple local optima

Other Notes

- As the number of participants in the market increases, generally these market dynamics will decrease.
- However, transmission system constraints can create a pocket of the system that may only be served by a small number of participants.
  - You would expect to see the same kind of behavior during these times.